Version 1.5 for Windows

RING

Theory and Modelling Guide

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Contents

Scope of guide

This document is designed to provide practical guidance on the application of RING 1.5 to the assessment of masonry arch bridges (RING 1.5 is computer software designed to compute the ultimate load carrying capacities of masonry arch bridges). The document also includes a brief description of the theoretical basis of the software, together with a description of validation studies which have been performed. For detailed step-by-step guidance on using the software, readers are referred to the companion RING 1.5 Program Reference Guide and/or the help file accessible when running the software.

1. Introduction

RING 1.5 is specialised software designed to compute the ultimate load carrying capacities of single and multi-span masonry arch bridges. RING has been publicly available since early 2001 and the software is now widely used by practitioners and researchers worldwide.

In order to apply RING 1.5 the user should have a basic understanding of the theoretical basis and range of applicability of the program. Hence this guide is designed to:

- explain the theoretical basis of RING 1.5;
- document studies undertaken to validate the results produced by RING 1.5;
- provide practical guidance on using RING 1.5 to assess masonry arch bridges.

This guide should be read in conjunction with the companion RING 1.5 Program Reference Guide, which can be obtained, along with the software itself, from http://www.shef.ac.uk/ring. The Program Reference Guide provides information on hardware and operating system requirements for RING 1.5, installation instructions and also detailed instructions on using software features.

2. Theoretical basis of RING 1.5

2.1 Background

RING 1.5 idealises a masonry arch structure as an assemblage of rigid blocks and uses *computational limit analysis* methods to analyse the collapse state only. Although limit analysis, or 'plastic'/ 'mechanism' analysis techniques were originally developed for steel components and structures, it has since been shown that these can be applied to masonry gravity structures, such as piers and arches[1].

To help understand why limit analysis theory is applicable, compare and contrast the response of a steel column with uniform plastic cross-section and a weakly mortared masonry pier, both subject to a horizontal load *F*, as shown on Figure 1.

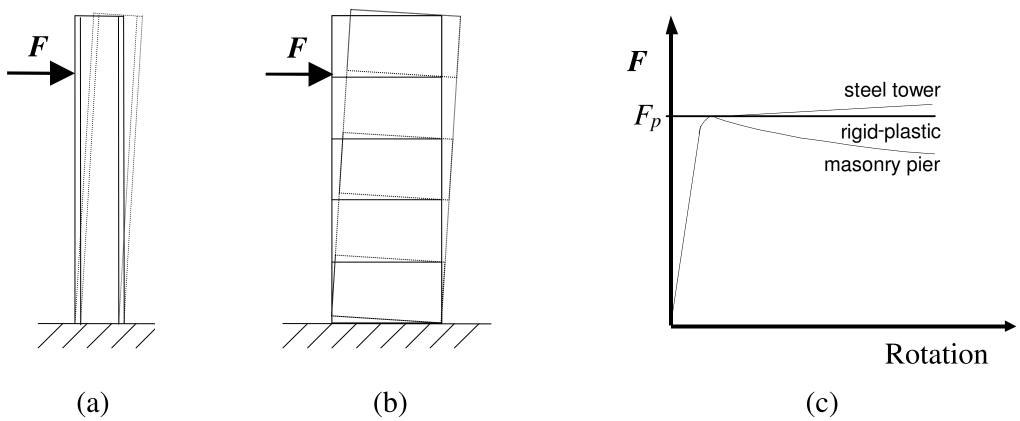


Figure 1 Laterally loaded (a) steel column, (b) masonry pier, and (c) idealised response curves

It can be deduced that:

- whilst the tensile and compressive strength of the steel column endow it with a finite plastic moment of resistance, M_p, the absence of tensile strength means that the masonry pier does not possess a comparable (i.e. strength derived) moment capacity;
- however, the thickness and self weight of the pier mean that there is some resistance against overturning and the masonry pier could conceptually be considered as possessing a moment capacity, albeit one that varies with height (equal in magnitude to the normal force at a given cross-section multiplied by half the pier thickness);
- furthermore, provided pier displacements do not become large, the resistance of the masonry pier against overturning at a given cross-section will remain broadly constant;
- hence the response of the pier can be considered 'ductile', an important requirement in order for limit analysis theory to be applicable.

As indicated above, in masonry structures the moment of resistance effectively varies continuously and consequentially this makes conventional bending moment diagrams difficult to interpret. Thus it is normally more useful to plot the eccentricity of the compressive force, or *thrust*, at each cross-section (where eccentricity = moment / thrust). The resulting *lines of thrust* at collapse for two different structural forms are shown on Figure 2.

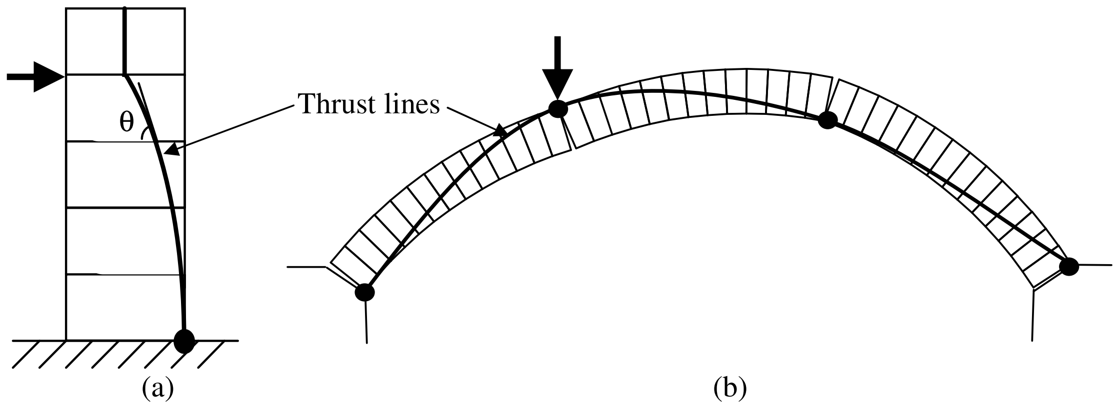


Figure 2 Thrust line at collapse in (a) masonry pier, and (b) masonry arch

Note that in the case of the masonry pier shown in Figure 2a statics alone may be used to uniquely determine the position of the thrust line both prior to and at ultimate failure (since the structure is *statically determinate*). In contrast, in the case of the masonry arch shown in Figure 2b there are many possible positions of the thrust line prior to failure and its position may only be uniquely determined at the point of ultimate failure (since the structure is *statically indeterminate*).

In addition to basic equilibrium considerations, in the context of masonry gravity structures, the following conditions may then be used to test for ultimate collapse (assuming both hinging[†] and sliding[‡] failures are considered possible):

(i) The *yield condition*, which may be deemed to be satisfied providing the line of thrust both lies entirely within the masonry and does not cross any joint at a subtended angle (θ) of less than $\tan^{-1}(\mu)$, where μ is the coefficient of friction.

(ii) The *mechanism condition*, which may be deemed to be satisfied providing the line of thrust either touches exterior faces of the masonry blocks and/or crosses sufficient joints at an angle (θ) of $\tan^{-1}(\mu)$ to create the releases required to transform the structure into a mechanism.

Thus in the context of masonry arches the lower bound theorem of plastic analysis can be stated as: *if a line of thrust satisfies the <u>equilibrium</u> and <u>yield</u> conditions, then the applied load will be a lower bound on the true plastic collapse load.*

Similarly the upper bound theorem of plastic analysis can be stated as: *if a line of thrust satisfies the <u>equilibrium</u> and <u>mechanism</u> conditions, then the applied load will be an upper bound on the true plastic collapse load.*

Using either an upper or lower bound approach, a hand limit analysis could be performed. For example, an upper bound hand analysis could be carried out by: (i) choosing a likely mechanism of collapse; (ii) using equilibrium (or the work method) to calculate the collapse load; (iii) trying other likely collapse mechanisms until the critical one has been found.

[†] Initially assuming that the masonry possesses infinite compressive strength, so the line of thrust can be transmitted through a hinge point lying on an exterior face of the arch.

[‡] Assuming that sliding failures obey an associative flow rule (i.e. 'sawtooth friction', where sliding is accompanied by dilatancy).

However, even in the case of a single span, single ring, arch their curved geometry makes such a hand based procedure particularly tedious. RING 1.5 may be considered to effectively automate this process, using rigorous mathematical optimisation techniques rather than a trial and error procedure to find the absolute minimum collapse load (refer to Annex A1 for details of the mathematical problem formulation).

Finally, whilst for sake of clarity the above discussion has implied that masonry possesses infinite compressive strength, this is clearly not the case in reality. In fact the presence of finite strength masonry means that the line of thrust mentioned previously is better thought of as a *zone of thrust*, which should have sufficient thickness at any point to carry the compressive force (the required thickness depends on the crushing strength of the masonry, which can be specified in RING). Note that in RING it is assumed that the thrust is carried by an area of material under a uniform level of stress (i.e. assuming a rectangular stress-block, in accordance with a rigid-plastic idealisation of the masonry crushing response).

2.2 The failure load factor

Although for simplicity the previous section considered a case where a collapse load (e.g. F_p) was to be computed, it is generally more useful to compute the factor which would, when applied to some specified pattern of live loads, leads to collapse. This factor (or 'multiplier') is commonly termed the 'load factor' and its determination for a given bridge is the principal goal of a RING analysis.

For example, if a 1kN single axle load is specified and RING indicates a computed failure load factor of for example 154, this means that the load which would cause collapse is 154kN. Alternatively if a 10kN single axle load was specified in the previous case the failure load factor computed would be 15.4 (15.4×10=154kN).

When the applied load comprises a series of axle loads, the failure load factor is the multiplier which, when applied to all axle loads simultaneously, just leads to collapse. For example, if a 1400kN rail vehicle comprises four 200kN axles and four 150kN axles and RING indicates a computed failure load factor of 3, this means that the loading at failure comprises four 600kN axles and four 450kN axles (3×200=600kN; 3×150=450kN).

Full details of the mathematical formulation are provided in Annex A1.

2.3 Range of applicability of RING 1.5

2.3.1 General

RING 1.5 is most suited to the analysis of short and medium span single and multi-span masonry arch bridges where foreseeable live loadings will typically be non-negligible in comparison to structural self-weight. There is no lower limit on the spans which can reasonably be analysed. For spans longer than 20 - 30m foreseeable live loadings will often be essentially negligible and other considerations become more important, such as long term masonry creep effects (due to persistent moderately high stresses). Additionally, in the case of very long span bridges the presence of high compressive stresses may give rise to non-negligible second order deformations, rendering RING 1.5 predicted carrying capacities potentially non-conservative.

RING 1.5 may be used to model bridges comprising single or multi-ring arch barrels constructed using regular stone blockwork or brickwork. Since the constituent blocks are assumed to be rectangular, the software is less well suited to modelling random rubble stone masonry arches.

Except for rigid-plastic crush zones at the hinges, RING 1.5 assumes the constituent blocks are rigid, and hence the software is not suitable for predicting the magnitudes of structural displacements prior to collapse. Additionally RING 1.5 is not suitable for accurately predicting the ultimate strength of a bridge if either of the following apply:

(i) The bridge comprises a long (e.g. > 20 - 30m) and/or flat arch (e.g. span/rise > ≈6, or the arch contains flat sections, for example in the case of an elliptical arch) and it can be expected that elastic deformations prior to collapse will significantly change the arch geometry;

(ii) A brittle response of some part of the structure may be expected to prevent the formation of a ductile collapse mechanism (e.g. abrupt failure of the bond between rings; brittle hinge formation).

However, even in these cases RING can provide an invaluable upper bound estimate of the likely strength of the bridge, which can be used as a benchmark for alternative analysis methods. Such alternative methods may comprise: in the case of (i) a geometrically non-linear elastic analysis; in the case of (ii) a non-linear elastic analysis incorporating a masonry material model respecting fracture mechanics principles (e.g. using Hillerborg's cohesive crack theory[2]).

Alternatively if in the case of (ii) the brittle response stems from shear failure at masonry joints, it may alternatively be assumed that the initial bond strength at these joints is zero, and that only normal compression and frictional forces may be transmitted. In this case RING 1.5 may be used and can normally be expected to provide lower bound (conservative) results.

Unfortunately the decision as to when cases (i) and (ii) might apply is complicated by the fact that both are stress level dependent. Thus if stresses are very low (in comparison to the elastic modulus and/or bond strength of the masonry), then brittle fracture of the bond between rings for example is unlikely to be an important issue. Thus since it can be shown that stresses increase with the size of structure being considered[†], this implies that brittle fracture of the bond between rings may not be of great concern in the case of very short span bridges; refer to section 4.9.2 for further discussion on modelling multi-ring arch bridges.

Additionally since RING 1.5 has been calibrated in situations when fill depths are relatively small in comparison to the arch span, for bridges with relatively small fill depths at the crown (or with no fill) RING 1.5 can be expected to provide reasonable results. Conversely, when the fill depth at the crown is large (e.g. > span/2) results from the program should be treated as being very approximate.

[†] Consider for example the stresses at the base of two geometrically similar solid masonry piers. If the second pier is n times as large, in all dimensions, as the first then its volume and hence self weight will be n^3 larger. However the area at the base of the pier will only be n^2 times larger, so it follows that the gravity stresses at the base (and elsewhere) in the second pier will be $n^3/n^2 = n$ times larger.

2.3.2 3D effects

In general spandrel walls at the edges of a bridge can stiffen the arch prior to failure and, depending on their end restraint conditions, may also enhance the ultimate limit strength. Studies of the influence of spandrel walls on the carrying capacity of full-scale single and multi-span laboratory bridges are detailed elsewhere[3,4].

However, if a bridge is wide in comparison to its span then the effects of the spandrel walls on the central section of the bridge may be quite minimal. Furthermore, a common defect observed in masonry bridges is detachment of the spandrel walls (this is evident by the presence of longitudinal cracks running close to the edges of the bridge). For these reasons spandrel walls are not modelled in RING 1.5.

Finally, since the software idealises the arch in two-dimensions, it is most suited for assessing masonry arch bridges which span squarely between abutments (section 4.5 provides a brief note on modelling skew bridges).

2.3.3 Range of collapse modes identifiable

The general problem formulation and rigorous mathematical optimisation solvers employed mean that RING 1.5 can identify numerous potential failure mechanisms. Figure 3 shows a selection of those which have been observed whilst using the program to assess real bridges.

The ability of RING to identify hitherto unknown failure modes has led to some interesting findings. For example common wisdom had it that if the piers are ‘stocky’, by whatever measure, then a multi-span bridge could safely be analysed as a series of separate single spans. However, the failure load factor associated with the mechanism shown on Figure 3f is actually much lower than that computed if the outer spans are omitted from the analysis - indicating that the presence of stocky piers (and backing in this case) is no guarantee that the structure can be safely idealised in this way. In general this indicates that users should where possible avoid using such rules of thumb and should model as much of the structure as is practicable (refer to section 4.6 for further advice on modelling multi-span bridges).

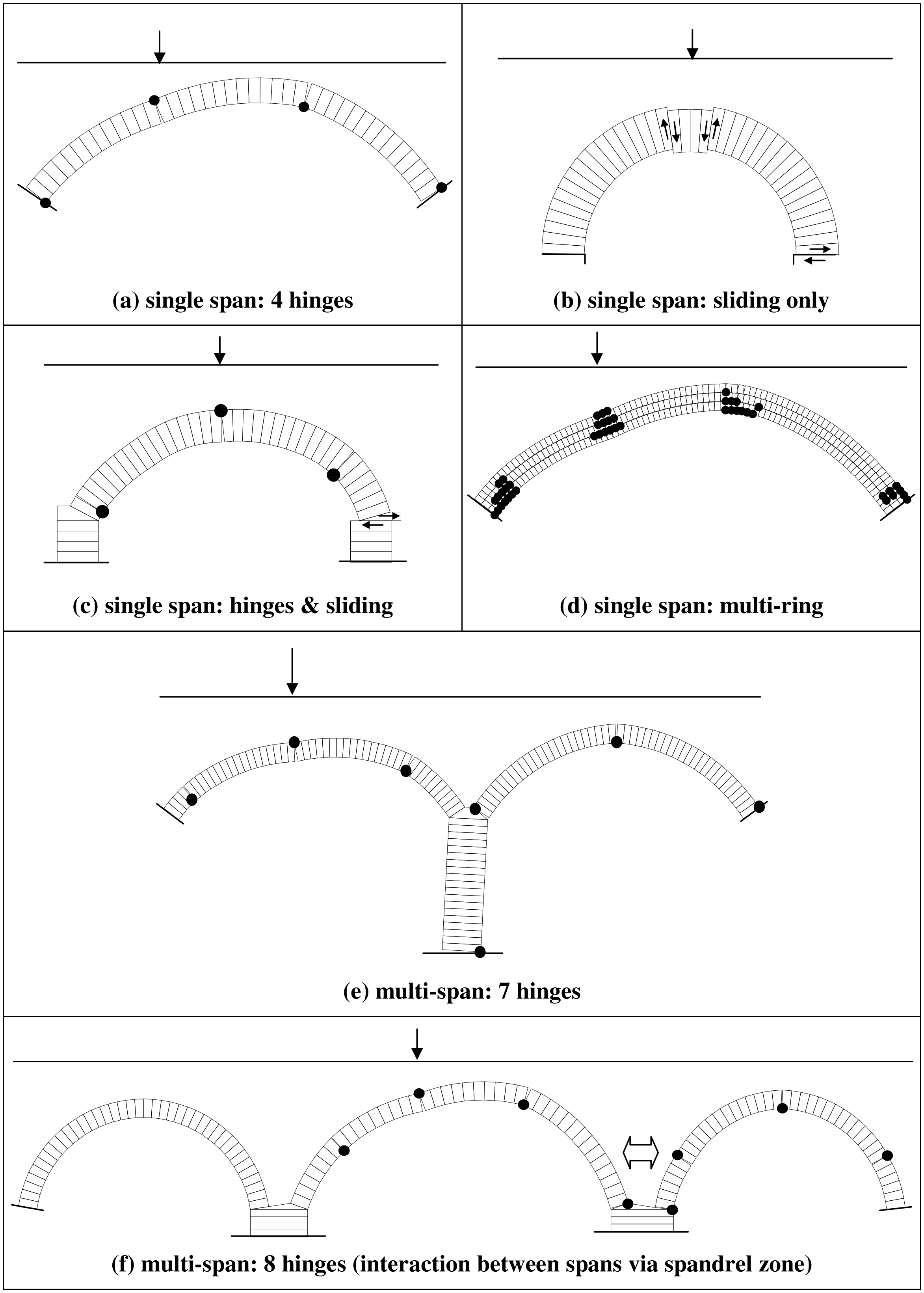


Figure 3 Selection of potential failure modes identified by RING

2.4 Modelling finite masonry strength

When finite material strength is specified, an iterative analysis is performed by RING 1.5, with the effective contact length at each contact being modified at each iteration according to the magnitude of the normal force. Note that it is assumed that the masonry in hinge zones crushes according to a rigid-plastic idealisation.

Convergence is assumed to be attained when the failure load factor computed at the current and last iterations are within a user-specified tolerance. The thrust in the arch is assumed to be transmitted by a rectangular stress block, and, using Livesley's approach[5], this means that at hinge positions the edges of the effective contact surfaces are modified at each iteration so as to coincide with the centre of the crush block (Figure 4). A stabilisation scheme is introduced to help avoid the cycling behaviour which sometimes prevents convergence using Livesley's basic approach; further details are provided in Annex A2.

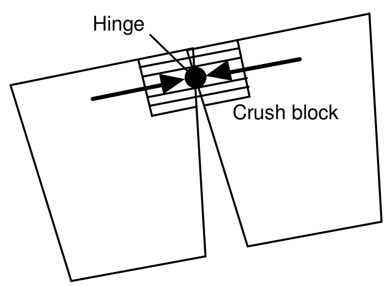


Figure 4 Location of hinge assumed (Livesley's approach)

2.5 Modelling sliding failures

Whilst many mechanism programs neglect the possibility that sliding failures might occur, this is not the case with RING 1.5. By default an associative (or 'saw tooth') model for friction is used (this means that separation is assumed to accompany sliding).

However it has been shown[6] that assuming associative (or 'saw tooth') friction between smooth blocks can lead to non-conservative load factors being obtained (if sliding is involved in the critical failure mode). Despite this, it has been found that when modelling multi-ring brickwork arch bridges reasonably good agreement between experimental and numerical results can be obtained (in fact it was found that the numerical multi-ring model always under-estimated the experimentally observed carrying capacity)[3].

2.6 Modelling backfill

2.6.1 General

The vertical dead weight of backfill material effectively pre-stresses the masonry in an arch, increasing its load carrying capacity (provided the constituent masonry has sufficient compressive strength). The backfill also has two other beneficial effects (Figure 5):

(i) It disperses live loads;

(ii) It can restrain movement of the arch when the latter sways into the fill. This is often termed 'passive' restraint.

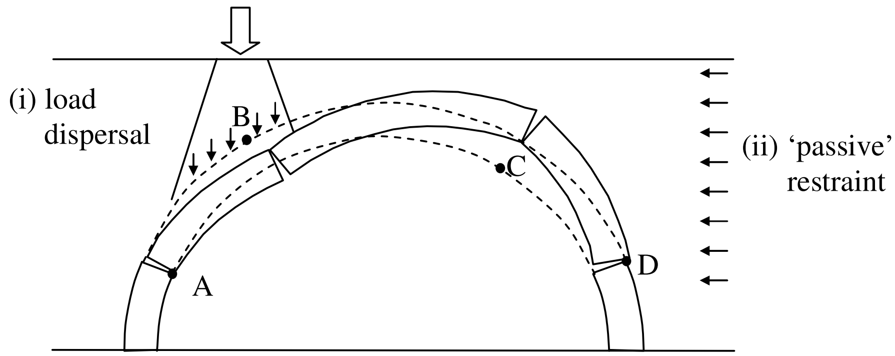


Figure 5 Masonry bridge soil-structure interaction

Each of the above effects can potentially significantly enhance the carrying capacity of a masonry arch bridge. However, whereas the constituent masonry blocks in masonry arch bridges are modelled explicitly in RING 1.5, the backfill is presently modelled in a more simplistic and indirect manner, as described in the following sections.

2.6.2 Dispersion of live loads

The vertical live load pressures on the back of an arch are assumed in RING 1.5 to either be: (i) uniformly distributed, the intensity being governed by the depth of fill under the centre of a given axle and the specified limiting dispersal angle; or (ii) to be dispersed according to a Boussinesq type distribution, with a limiting dispersion angle being specifiable by the user.

Note that from a theoretical perspective the use of a Boussinesq type distribution is not entirely satisfactory (since (i) there does not exist a semi-infinite elastic half space below the load and (ii) the elastic distribution indicated may be seen as being incompatible with an ultimate load analysis). Nevertheless the distribution has been used for many years in masonry arch analysis programs (e.g. Choo et al[7]) and laboratory tests have indicated that the Boussinesq distribution provides a better approximation of reality than uniform pressure distributions, and also models the effects of overlapping dispersed loads more appropriately. This is therefore the default model in RING 1.5.

2.6.3 Passive restraint

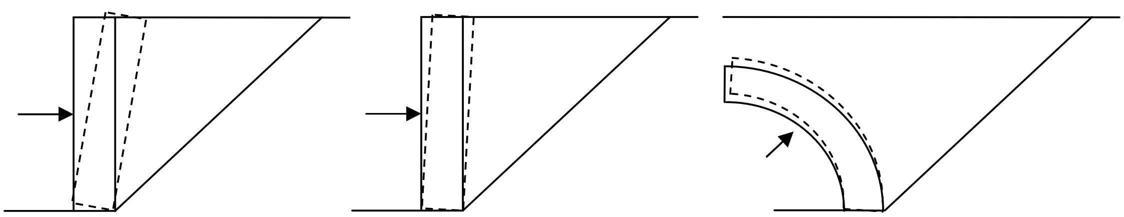
To date classical vertical retaining wall theory has often been used to provide an approximation of the restraining pressures mobilised when a section of arch pushes into the surrounding fill material.

Classical vertical retaining wall theory indicates that the horizontal passive restraining pressures can be computed by multiplying the vertical weight stresses by an earth pressure coefficient K_p, (Figure 6a) where:

$$K_p = \tan^2\left(45 + \phi'/2\right)$$

and where ϕ' is the effective angle of shearing resistance of the fill material.

In the case of failure involving wall rotation, since full passive pressures are not mobilised unless rotations are very large, a reduced earth pressure coefficient is often used in practice (Figure 6b). If the wall is actually curved (i.e. part of an arch) then the coefficient is often assumed to be further reduced (Figure 6c).



(a) 1.0 K_p **(b) ½ K_p** **(c) ⅓ K_p**

Figure 6 Commonly used horizontal earth pressure coefficients: (a) large wall rotation, (b) small wall rotation (≈ 0.5%), (c) arch segment rotation into surrounding fill

Using a reduction factor of ⅓ leads to a restraint force which is approximately equal to that measured in laboratory tests (at least for frictional backfills with a high value of ϕ'), and hence is recommended for use in RING 1.5. However, it should be appreciated that this factor is purely empirical, and has not been derived theoretically (e.g. by considering the kinematics of arch-soil interaction).

Burroughs et al.[8] have recently suggested instead using an alternative pressure coefficient for arches:

$$K_e = K_0 + e(K_p - K_0)$$

where K_0 is at rest earth pressure coefficient (typically taken as 1 - sinϕ), and e is an empirical factor. When ϕ' is high and e is taken as ⅓ then the pressure coefficient $K_e \approx K_p/3$ (since K_0 is very small compared with K_p). Alternatively, when ϕ' is small, $K_e > K_p/3$. However, such an increased coefficient should only be used with great caution in RING 1.5. This is because destabilising active earth pressures are by default not included in the software but these can start to become significant in the case of backfills with low values of ϕ'. Hence at the present time it is not recommended that Burroughs's coefficient K_e is used in RING 1.5.

In RING 1.5 in addition to the 'classical' triangular distribution of pressures implied in the previous discussion, a 'uniform' distribution is also available for use in special circumstances (the 'classical' model is the default).

Recent research suggests that the cohesive strength of clayey backfill materials may significantly enhance the strength of short span bridges. However, at present more research is required before advantage of this can be taken on a routine basis.

2.6.3.1 Implementation in RING 1.5

It has been found that a simple but effective method of incorporating horizontal fill pressures is to introduce so-called fill elements (Figure 7).

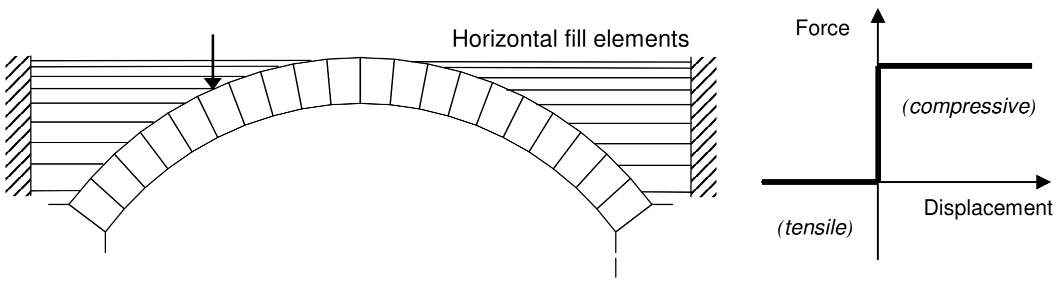


Figure 7 Arch restrained with uniaxial fill elements (a), and (b) fill element response

These fill elements compress at constant force (e.g. $K_p \times$ factor $\times$ vertical pressure $\times$ area), but are unable to extend. This ensures that pressures are mobilised in the correct sense ('active' pressures are usually relatively small, and so are by default neglected in RING 1.5).

Note also that an upper limit is placed on the magnitude of the horizontal backfill pressures that can be applied to a given masonry block without causing the strip of backfill on the block to slide (this can be overridden if a user-defined pressure is specified).

2.6.3.2 Simplifications inherent in current approach

There are several important simplifications inherent in the way RING 1.5 treats passive (and active) restraining pressures: (i) passive backfill pressures are assumed to be mobilised by infinitesimal structural movements; (ii) active pressures are neglected; (iii) unusual failure modes may . Further background information on these assumptions is provided in Annex B.

2.6.4 <u>Masonry Backing</u>

Horizontally aligned uniaxial elements are also used to simply model the influence of masonry backing (if present). Note though that: (i) the magnitude of the force required to compress a backing element is by default set sufficiently high so that the backing generally behaves as a series of rigid struts in compression (the default limiting compressive stresses can be edited if necessary), and (ii) the magnitude of the limiting compressive force is not limited by backing/arch sliding considerations, as is the case with backfill elements (see section 2.6.3.1). This latter assumption implies that relative sliding between the arch and backing cannot occur. If this is not known to be the case then backing should only be safely included at positions where the angle between the line drawn tangentially to the arch extrados and the horizontal is suitably large.

3. Validation of RING 1.5 against full-scale bridge tests

3.1 Bolton laboratory tests

At Bolton Institute, UK, in the early 1990's a number of 3m and 5m span bridges were tested in the laboratory. A key advantage of these tests over field tests (e.g. see section 3.2) was that the internal constructional details and material properties were known. RING was originally developed to assist with the interpretation of the results from these laboratory tests.

Since the original publication of the work in The Structural Engineer[3,4,9], the program has been enhanced to include, amongst other things, material crushing and more realistic models of the dispersion of the applied load through the backfill. In Table 1 sample updated RING 1.5 analysis results are presented alongside experimental test results (only bridges with detached spandrel walls are included since these behave in a two dimensional manner). To obtain these latter RING results a classical earth pressure coefficient was specified (rather than the back-substituted experimentally recorded pressures, as used in the original publications), with the theoretical passive pressure coefficient derived from vertical retaining wall theory factored by ⅓ (see section 2.4.3). It is clear from Table 1 that, using a single effective earth pressure coefficient, reasonable results can be obtained for a variety of different bridge geometries.

Bridge	Description	Expt. collapse load (kN)	RING 1.5 analysis			Theoretical /experimental collapse load
			Limiting load dispersion angle (degrees)	Effective classical passive earth pressure coefficient	Theoretical collapse load (kN)	
3-1	3m single span	540	45	4.5*	550	102%
3-2	3m single span; debonded arch rings	360	45	4.5*	245	68%
5-1	5m single span	1720⁺	45	4.5*	2238	130%⁺
5-2	5m single span; debonded arch rings	500	45	4.5*	463	93%
Multi-2	3m triple span	320	45	4.5*	358	112%

*approx. ⅓ of the full classical passive pressure coefficient indicated by the measured ϕ' value of 60°

⁺the experimental collapse load of this bridge was reduced by the sudden onset of partial ring separation

Table 1 Sample comparison between Bolton laboratory and RING 1.5 collapse loads

The RING 1.5 data files of the above runs are distributed with RING (these are located in the 'Samples' subdirectory, e.g. C:\Program Files\RING 1.5\Samples).

3.2 Field bridge tests

In the late 1980's and early 1990's, the Transport and Road Research Laboratory (TRRL, now TRL) carried out a series of load tests to collapse on redundant arch bridges. Most bridges failed in four hinge mechanisms, although some of the bridges were reported as failing by 'three hinge snap through' or in 'compression' (material failure). It was likely that many of the bridges tested were restrained considerably by their attached spandrel walls and/or masonry backing. Outline information on these bridge tests has been provided by Page[10].

With the benefit of hindsight, significantly more pre-test investigation work should have been performed to better characterise the internal construction details and material properties. This would have been useful in providing a more comprehensive data set for use by analysts who have since attempted to model the behaviour of the bridges under load.

In 2001 TRL were commissioned to independently validate RING 1.1 and other available masonry arch bridge analysis software. Despite the uncertainties outlined above, as part of the validation process it was decided that the programs would be used to predict the carrying capacities of 5 of the field bridges load tested more than a decade previously. Details taken from the TRL report[11] relating to RING are provided in Table 2 below.

Bridge[†]	**Theoretical /experimental collapse load**
Torksey	81%
Bridgemill	100%
Barlae	92%
Preston	90%

Table 2 Correlation between TRL field bridge test and RING collapse loads (independently produced by TRL)

It is evident that agreement between the RING predictions and the full-scale test results was found to be reasonably good. Thus the TRL report concluded that RING 'gives good results' and that 'RING, with some investment in an improved solver, would be a very effective tool for most assessment engineers….'. The concern about the slow speed of the solver was largely addressed following the release of RING 1.5 which is up to 200× faster than RING 1.1 which was used in the 2001 TRL study (N.B. the collapse load factors computed by RING 1.1 and RING 1.5 are essentially identical).

Based on this evidence Network Rail have confirmed that RING is a suitable program for use to assess masonry arch bridges on the rail network.

[†] Strathmashie bridge was also modelled but was in poor condition and, because 'none of [the] defects were modelled during the analysis, all the programs returned non-conservative results'.

4. Using RING 1.5 in practice

4.1 Parameter selection

The first step in an analysis is typically to identify sensible values for the analysis parameters. Default values are provided but it should be borne in mind that some of these default values may be quite conservative; in all cases more accurate values should be used where possible.

Additionally, in order to save computational effort a default value of 40 for the number of blocks per arch ring is specified (which is likely to be different to the actual number of units). This may lead to a very small overestimate of the predicted carrying capacity of a given bridge if the real structure contains more than 40 blocks (e.g. up to a few percent).

4.2 Partial Factors of Safety

RING 1.5 does not allow partial factors of safety to be specified; hence these must be applied to material properties prior to analysis, as deemed appropriate by the user.

Some assessment codes use a global condition factor. A global condition factor should not normally be applied to RING 1.5 analysis output as the effects of defects such as ring separation, low strength masonry and the influence longitudinal cracks have on the ability of a given bridge to distribute the load transversely may all be accounted for directly.

4.3 Loading

4.3.1 Loading models

User-specified loading models may be set up by the user. Typically the most onerous loading pattern for most small to medium span masonry arch bridges will be the presence of single (or twin) axles on an arch.

Some loading models contain components of distributed loading: given the origins of such load models (typically determined from influence lines for simply supported or continuous beams) it is debatable as to how appropriate their use is for masonry arch bridges. One practical issue is that the length of distributed loading may be variable, and in this case *it is the responsibility of the user to determine the length of distributed loading which proves to be most onerous.*

4.3.2 Longitudinal dispersion of load through fill

Live load is spread through the fill according to a user specified model (uniform or modified Boussinesq distribution); refer to section 2.6.2.

4.3.3 Transverse distribution & effective bridge width

RING 1.5 is a 2D analysis program. Thus the user needs to make appropriate assumptions in order to determine the effective width of bridge which may be assumed to support an axle loading.

Unfortunately this is an area for which there is little real evidence on which to base rational rules. Typically codes of practice suggest calculating an effective bridge width by assuming a prescribed amount of transverse load spread under the load. However an effective bridge width so calculated may not be reasonable and the user should check whether for example longitudinal cracks in the arch barrel, the proximity of adjacent track or the edge of the bridge will limit the effective width further.

Other effects (e.g. centrifugal action) may mean that one wheel applies more load to the structure than the other; this may mean that a concentrated wheel loading becomes the critical case and hence that a reduced effective width should be selected. See section 4.3.5.1

4.3.4 Dynamic effects

In many cases codes specify that dynamic factors are applied to all loads simultaneously. This is important as it means the load *pattern* remains unchanged, and hence that dynamic effects only need to be considered after a RING 1.5 analysis has been completed.

In cases when different dynamic factors are applied to different axles then the resulting change to the load pattern means that this needs to be taken account of in a RING 1.5 analysis. In practice this means that several variants of a basic load model should be set up, one created each time the relevant dynamic factor is applied to a different axle; *it is the responsibility of the user to determine which variant of the load model proves to be most onerous.*

4.3.5 Other effects

4.3.5.1 Nosing and centrifugal forces

On horizontally curved roads (or railway tracks) the vertical effects of centrifugal and nosing[†] actions can lead to one side of the carriageway (or one rail) being more heavily loaded than the other. Both actions are speed dependent.

In either case it is usually considered prudent to consider wheel load cases separately from axle load cases, to determine whether or not these are critical. In RING 1.5 this requires the use of a suitably reduced effective width (see section 4.3.3). However, since the *pattern* of loading is unaffected then this special load case can normally be considered retrospectively (i.e. after an analysis has been performed, by modifying the failure load factor to account for the use of a different effective width and live load intensity).

4.3.5.2 Traction/braking forces

In RING 1.5 it is currently unfortunately not easy to apply horizontal forces at road / rail level (e.g. to model traction/braking forces). This is principally because the current simplified model of the backfill was developed for vertical loads and is not readily modifiable to account for horizontal loads.

[†] Nosing forces are caused by side contact of the wheel flange on a rail. Since the forces are generally assumed to act perpendicular to the rail, on canted track there will be a small vertical component to consider (applied to one rail only).

However, it is possible to apply user-specified horizontal forces (as pressures) directly to blocks within arches and/or piers; refer to the companion RING 1.5 Program Reference Guide for further details.

4.4 Modelling the shape of the arch

It is the shape of the arch in relation to the pattern of loadings applied to it which governs stability, and hence it is of paramount importance that due care is taken when recording and entering the shape into RING 1.5. All too often this is ignored, with the default 'Segmental' arch shape often being unthinkingly used[†]. When transverse cracks are present it follows that the arch profile must differ from that originally constructed, and this makes it even more important to perform an accurate survey of the bridge prior to analysis, to ensure that the true shape of the arch is modelled.

In RING 1.5 the arch can either be modelled as 'Segmental' or as 'User-defined'. When modelled as 'Segmental' the software models the arch profile as a single circular line segment whereas when modelled as 'User-defined' the software attempts to fit a series of segmental line segments between the points given, using the assumption that one of the points corresponds to the maximum arch rise. Sometimes some manipulation of raw survey data is required to ensure this is accepted by the software (particularly if relatively inaccurate survey data is used).

To obtain an indication of the influence of arch shape on carrying capacity, refer to Figure 8, which shows the effect on the computed load factor of simultaneously modifying the ¼ and ¾ point rises of Bolton bridge 3-1 (critical load position allowed to vary). In this case the maximum computed load factor approximately corresponds to the segmental arch shape, and it is evident that quite significant reductions in load factor are observed as the shape deviates from this (span and crown rise fixed).

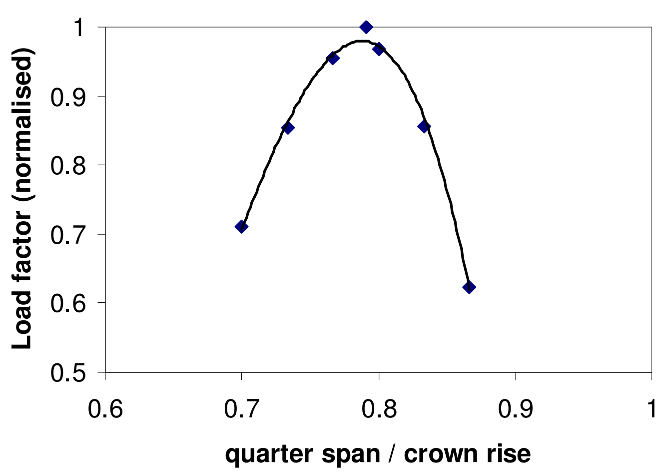


Figure 8 Influence of varying ¼ and ¾ point rises (as ratio of midspan rise)

[†] Indeed it should be considered just as important to accurately record the arch shape and thickness as it would, for example, be to accurately measure the overall depth, flange thickness etc. of a steel I-beam prior to analysis.

It is also important to note that if the arch shape is asymmetrical (i.e. if the quarter and three-quarter point rises differ) then carrying capacity will often be significantly reduced.

4.5 Skew bridges

Since RING 1.5 is 2D analysis software, it is most suitable for the analysis of bridges which span squarely between abutments. Skew bridges tested in the laboratory[12] have exhibited distinct 3D failure modes which cannot be replicated using a 2D analysis tool.

However, given the comparative computational expense and lack of availability of mainstream 3D analysis tools, some codes of practice pragmatically permit the use of 2D analysis methods for skew bridges; in such cases whilst the code requirements can be followed the approximate nature of the results so obtained should be appreciated.

4.6 Multi-span bridges

Multi-span bridges are modelled in RING 1.5 in exactly the same way as single-span structures, i.e. in essence simply as assemblages of interacting rigid blocks and backfill elements. Using RING 1.5 the most critical failure mode will automatically be identified, whether this involves a single or multi-span failure mode. Multi-span failure modes typically, although not always, involve two adjacent spans (although note that for a bridge with slender piers initial failure of one or two spans is likely in practice to quickly precipitate failure of neighbouring spans, because of the out-of-balance thrusts which then act at the tops of piers supporting these).

Additional notes on modelling multi-span arch bridges:

- for a viaduct comprising a large number of identical spans only two representative adjacent spans need normally be modelled initially;
- large railway viaducts frequently had large ‘king piers’ at frequent intervals (e.g. every 6 spans). These are typically sufficiently massive to ensure that no interaction occurs between the two spans abutting a ‘king pier’.
- for a bridge comprising spans of different geometries, ideally all spans between ‘king piers’ should be modelled;
- full-scale laboratory tests indicate that significant backfill pressures can be mobilised above the piers between adjacent spans, enhancing carrying capacity[4];
- alternatively, backing or internal spandrels are often present between spans, and this can play an important role in propping apart adjacent spans;
- in some cases the presence of strong fill or backing above a pier may mean that adjacent spans can interact in the failure mechanism without movement of the intermediate pier (e.g. see Figure 3f);
- in cases where intermediate piers are very slender, the user should consider separately performing other local checks (e.g. that elastic instability will not limit the vertical load that can be applied; no such check is currently done by RING 1.5).

4.7 Freestanding abutment blocks

This feature was originally included to allow modelling of a number of the full-scale model multi-span bridges and also a field bridge with an arch span adjacent to a beam span (Rotherham Road[13]). It was not originally intended that the feature would be used to model 'normal' abutment blocks (which have vertical faces in contact with backfill). Note in particular that:

- Although with care it is possible to model abutment blocks which have vertical faces in contact with backfill, as noted previously, horizontal pressures given by ⅓Kp are likely to under-estimate those which would be mobilized by an abutment sliding failure. It is hence reasonable to assume that the arch springs from rigid abutments (a recent laboratory test verifies this, although more research data is ideally required).
- Furthermore, whilst normally some or all of a live load which is positioned at (or beyond) the extremity of a bridge may 'miss' the structure, and hence is neglected, when abutment blocks are present this is not the case; instead whatever live load is detected on the structure is automatically factored up by RING 1.5 to ensure that no live load is lost. If this model does not match reality then the load capacity determined may be significantly over-conservative.

4.8 Horizontal backfill restraint and 'backing'

Except in the case of relatively shallow arches, the restraint offered by fill material behind an arch can lead to very significant increases in carrying capacity. The problem lies in determining, for the purposes of analysis, what level of restraint is likely to be available.

In the Bolton tests described earlier, soil pressure cells were installed at varying depths within the soil mass above both springings. Although results from the cells did not indicate either a classical triangular horizontal soil pressure distribution or that the total restraint was equal in magnitude to that indicated using classical theory, very significant horizontal fill pressures were recorded behind the side of a single-span arch swaying into the fill. Horizontal fill pressures at the other (active) side of the arch typically tended to quickly reduce as movements commenced.

A further problem is that where as in practice significant restraining pressures may only be realised when movements of the arch have become quite large, a conventional mechanism analysis works with infinitesimal displacements and so it generally has to be assumed that significant horizontal pressures are effectively present from the outset[*].

Due to the uncertainties outlined above, relatively low values for the effective coefficient of passive pressure should generally be used in analysis [e.g. effective passive coefficient = $\frac{1}{3}K_p$]. To calculate the K_p value to be used the angle of friction of the fill material must be assessed; it can normally be assumed that this will be at least 30º [when $\phi' = 30º$, $K_p = 3$ and hence $\frac{1}{3}K_p = 1$, which is the value of the coefficient used by default in RING 1.5]. To justify the use of higher values it is likely to be appropriate to perform appropriate intrusive investigations (e.g. dig trial pits, perform penetrometer testing etc.). Such investigations can also be useful in identifying unexpected beneficial construction details (e.g. generous concrete backing).

[*] However, compaction pressures, which are only present at depths less than approx. 3m, are of particular interest in the case of short span bridges since these can be considered to be mobilized without gross movements of the arch into the fill.

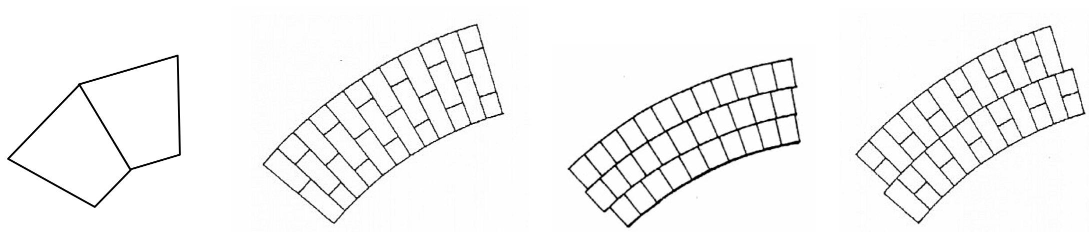
4.9 Modelling defects

4.9.1 Missing mortar and/or localised spalling of masonry units

Localised areas of missing mortar or occasional spalled masonry units are inevitable in old masonry structures, and need not be considered a cause for concern. However in cases when mortar loss and/or spalled units are more widespread and it is hence considered that the effective thickness of the arch barrel is being tangibly reduced then this should be accounted for in the analysis. In RING 1.5 localised mortar loss etc. cannot easily be modelled[†]. Hence this is normally modelled by globally reducing the arch ring thickness.

4.9.2 Ring separation in multi-ring brickwork arches

Various bonding styles are used in the arch barrels of masonry arch bridges (Figure 9).



(a) stone voussoir [1 ring] (b) brick: header [1 ring] (c) brick: stretcher [3 rings] (d) brick: hybrid [2 effective rings]

Figure 9 Typical masonry arch bonding patterns

It is usually satisfactory to model a brickwork arch with a bonding pattern of the form shown in Figure 9b as a single ring voussoir arch, as shown on Figure 9a. However, if very weak bricks are present headers may shear through and the arch should then be modelled as if composed from separate rings; e.g. see Figure 10 for a real-world example of this.

(a) **(b)**

Figure 10 (a) Arch barrel containing headers which have sheared through; (b) view through core hole in same bridge showing separation of bottom layer of brickwork

[†] There is a non-standard method for making localised changes to the ring thickness: (i) specify the arch geometry as 'user defined'; (ii) manually edit the intrados co-ordinates of the joints. These are shown in the .rng file (under the USER_SPAN_DETAILS heading) and may be edited using a text editor. However, changes made in this way will be lost once any subsequent change is made using the normal RING property editor.

However, even when inter-ring cracks are not evident the circumferential mortar joints present in the multi-ring bonding patterns shown in Figure 9c and 9d form potential surfaces of weakness and careful consideration should be given to modelling bridges constructed with these types of barrels.

In fact, when multi-ring brickwork arch bridges are assessed, the general question arises: should, for the purposes of analysis, the individual rings in the arch barrels of these bridges be assumed to be adequately adhered together or not? From the Bolton work referred to in section 3.1 the following was observed:

- Two 5m span arch bridges containing 440mm thick arch barrels in which individual rings had been properly mortared together did suffer ring separation whilst being load tested to collapse. This separation prevented the bridges from reaching the collapse load they otherwise would have (the sudden onset of ring separation was estimated to have reduced the respective carrying capacities of the two bridges by 20% and 55%).
- A similarly constructed 3m span arch bridge containing a mortar bonded 215mm thick arch barrel did not suffer ring separation when loaded to collapse

In addition to the in-situ mechanical properties of a given joint (the above bridges were constructed using a 1:2:9 [sand:cement:lime] mortar), scaling effects will also be important in governing the likelihood of ring separation. Essentially, by almost doubling the span, rise etc, the internal stresses will also be almost doubled and for this reason very short span bridges are likely to be less susceptible to ring separation than bridges with longer spans. This can be taken account of in the analysis (e.g. a 2m span multi-ring brickwork arch in good condition may justifiably be analysed with fully bonded rings but this is unlikely to be an appropriate idealisation for a geometrically similar 20m span bridge – since the higher stresses would mean it would be likely to suffer ring separation were the bridge to be loaded to collapse).

4.9.3 Cracking in the arch barrel

4.9.3.1 Macro cracks

There are several distinct types of macro-cracks observed in masonry arch bridges: e.g. longitudinal, transverse and diagonal. The potential influence of longitudinal cracks on the effective bridge width and hence on ultimate carrying capacity has already been briefly discussed in section 4.3.3.

Transverse cracks may be caused by small movements of the supports (e.g. perhaps due to slight outward spreading following decentering). Identification in an arch of transverse cracks indicative of formation of three hinges is not necessarily of concern (provided the abutments are sound); this is simply the statically determinate form of an arch. However, if the location of the crown region crack/hinge is observed to change under the action of normal traffic loading then this can be problematic, with subsequent fatigue failure of the structure a possibility.

Less frequently transverse cracks may be identified which are indicative of the partial formation of hinges due to excessive live loading at some point in the past. In general when sufficient releases (hinges and/or sliding planes) to form a mechanism are identified this should be considered to be potentially very serious. Whilst it is theoretically possible that there might exist additional reserves of strength, for example because of the potential for increasing passive soil restraint to be mobilised as structural movements increase, in this case the large structural deformations required are such that the structure will anyway fail to meet any meaningful serviceability criteria.

It may be feasible to point up transverse cracks and in this case a RING 1.5 analysis may be performed. Otherwise, since it is not easy in RING 1.5 to locally reduce the arch thickness at the position of the crack, and since RING 1.5 is not capable of modelling the presence of cracks which close up after finite movement, another analysis tool should be selected.

In square spanning bridges diagonal cracks may often be caused by abutment settlements. Whatever the cause, when diagonal cracks are present the arch profile should be surveyed at several positions across the width of the bridge, with an analysis being performed for each profile.

4.9.3.2 Micro cracks

Isolated fine cracks may be present in masonry joints or within masonry units. When numerous cracks are concentrated in parts of the structure, this can be indicative of a major problem, especially if there are signs of recent cracks. This is because masonry tends to fail in compression due to tensile splitting of the masonry units and hence widespread micro-cracking may indicate that the compressive capacity of the masonry has been almost exhausted (note that rough calculations of the stresses within a masonry pier can be misleading because the pier may be hollow or rubble filled). The cracks may continue to propagate due to long-term creep effects or due to the effects of repeated (fatigue) live loading. Masonry compression failures, although rare, will generally have catastrophic consequences and thus the assessment of a bridge with such symptoms should be carried out with extreme care; a simple RING 1.5 analysis alone is highly unlikely to be appropriate in such a case.

5. References

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INFORMATIVE ANNEX A: Mathematical formulation

A1 Joint equilibrium formulation

This section contains details of the mathematical formulation used in RING 1.5. This is slightly different to that used in RING 1.1[‡]. A major reason for using a different formulation was to improve numerical efficiency for multi-ring problems which were originally slow to solve (N.B. there have been no major changes in the basic assumptions made, i.e. elastic strains are negligible, blocks initially fit perfectly together etc. A minor change is that the inter-ring joint contacts in multi-ring problems are now treated in the same way as all other contacts; previously an inter-ring joint was modelled using a series of point contacts).

Currently a joint equilibrium formulation, similar to that proposed initially by Livesley[A14] is used[†]. Whilst this formulation produces a large number of constraints and variables, the total number of non-zero elements will generally be relatively small, which means that it can be solved very efficiently using modern interior point Linear Programming (LP) algorithms.

Thus assuming there are b blocks and c contact surfaces, the problem may be stated as follows:

$$\text{Max } \lambda$$

subject to:

$$\mathbf{Bq} - \lambda \mathbf{f}_L = \mathbf{f}_D$$

$$\left.\begin{array}{l} m_i \le 0.5 n_i t_i \\ m_i \ge -0.5 n_i t_i \\ s_i \le \mu_i n_i \\ s_i \ge -\mu_i n_i \end{array}\right\} \quad \text{for each contact, } i = 1,\ldots,c \qquad (1)$$

where λ is the load factor, **B** is a suitable ($3b \times 3c$) equilibrium matrix containing direction cosines and **q** and **f** are respectively vectors of contact forces and block loads. Thus $\mathbf{q}^{\mathbf{T}} = \{n_1, s_1, m_1, n_2, s_2, m_2, \ldots n_c, s_c, m_c\}$; $\mathbf{f} = \mathbf{f}_D + \lambda \mathbf{f}_L$ where $\mathbf{f}_D$ and $\mathbf{f}_L$ are respectively vectors of dead and live loads. Contact and block forces, dimensions and frictional properties are shown on Figure A1-1. Using this formulation the linear programming problem variables are the contact forces: n_i, s_i, m_i (where $n_i \ge 0$; s_i, m_i are free variables).

[‡] RING 1.1 used a redundant forces formulation (or, more precisely, a dual redundant forces formulation), which produced a very compact LP constraint matrix (or 'tableau'), but with a high proportion of non-zero elements.

[†] More precisely, RING 1.5 uses a dual joint equilibrium formulation (using the work method). However the primal and dual formulations are mathematically equivalent and for clarity the primal formulation is presented here.

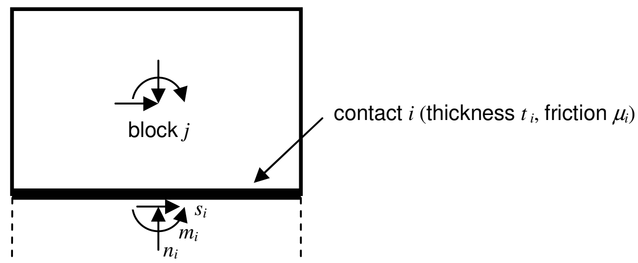

Figure A1-1 Block and contact forces

A2 Finite masonry material strength

At *each* joint the minimum depth of crushed masonry required to carry the normal force n at that joint may be calculated. Assuming a rectangular crush block with crushing strength σ_{crush}, and arch width b, this depth is:

$$d_{crush} = \frac{n}{\sigma_{crush} b}$$

Using the method proposed by Livesley the effective ring thickness of the arch is reduced by $d_{crush}/2$ at all joints, where d_{crush} is calculated using the expression above. The hinges in the subsequent analysis are then free to form on modified intrados and extrados surfaces [e.g. a given extrados hinge in a subsequent analysis will be forced to form at a depth of $d_{crush}/2$ as shown on Figure 4]. One potential problem with the method is that cyclic behaviour is sometimes observed, with large differences in the calculated thrusts in each of the rings being observed in consecutive analyses. To overcome this, the depth of crushing at a given iteration i may alternatively be computed as:

$$d^{i}_{crush} = \beta\left(\frac{n}{\sigma_{crush} b}\right) + (1-\beta)\left(d^{i-1}_{crush}\right)$$

Where superscripts i and i-1 indicate the current and previous iteration respectively, and β is a convergence parameter which can range in value between 0 and 1. As β is reduced from 1, it is found that convergence becomes progressively slower, but more reliable. In RING 1.5/2.0, β is taken as 0.6.

Figure A2-1 provides an example where cycling occurs when RING 1.1 is used but an improved convergence response occurs under RING 1.5 (multi-ring arch example with very low strength masonry).

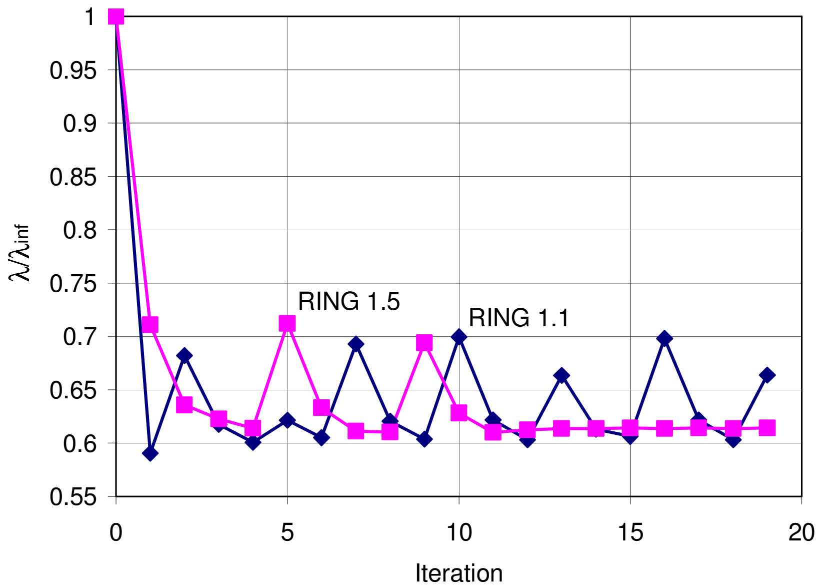

Figure A2-1 Sample convergence characteristics for a multi-ring arch problem

It is evident from Figure A2-1 that convergence is not monotonic. It is evident that although cycling occurred in the case of RING 1.1, a conservative estimate of the load factor could have been obtained by using taking the lowest load factor computed during the course of the iterative analysis procedure; this has found generally to be the case.

References

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INFORMATIVE ANNEX B: Notes on the backfill pressure model

B1 Gradual build-up of backfill pressures

As in practice peak fill pressures are only mobilised when structural deformations are large, it can be argued that a gross displacement analysis[B15] is required (a normal RING 1.5 analysis assumes infinitesimal deformations). However, the fill in most short span bridges will be very well compacted (due to trafficking) and hence quite large pressures can be expected to be mobilised even when displacements are relatively small. For example Figure B-1 shows the mean horizontal pressures mobilised above the springings or piers of three of the most well instrumented bridges tested at Bolton Institute in the 1990's (refer also to section 3). These bridges were backfilled with a well compacted graded crushed limestone fill material.

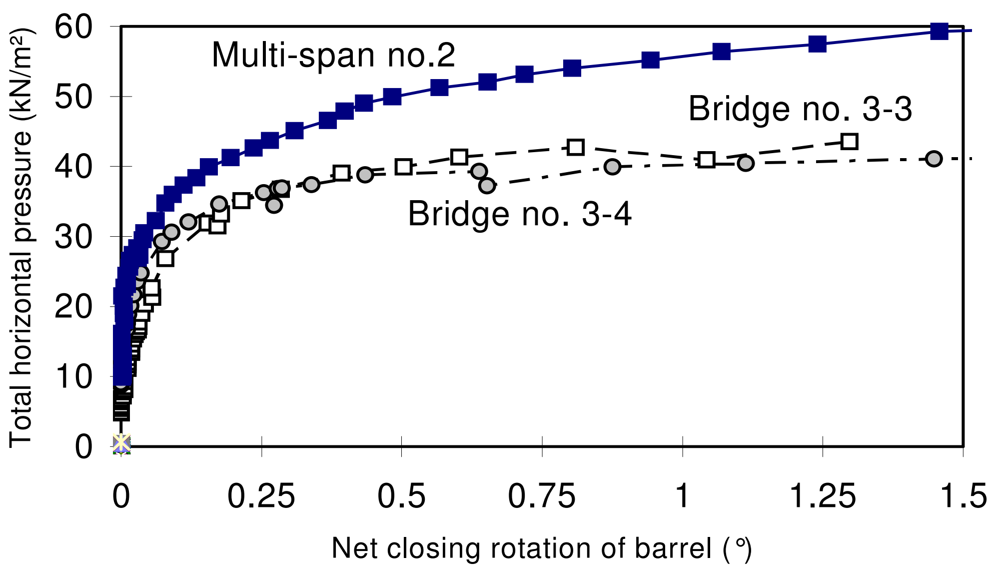

Figure B-1 Mean horizontal backfill pressure vs. net closing barrel rotation of arch barrels

A special version of RING was used to perform a gross displacement analysis, with the experimentally observed build up in pressures back-substituted into the analysis. The trend shown in Figure B-2 was obtained.

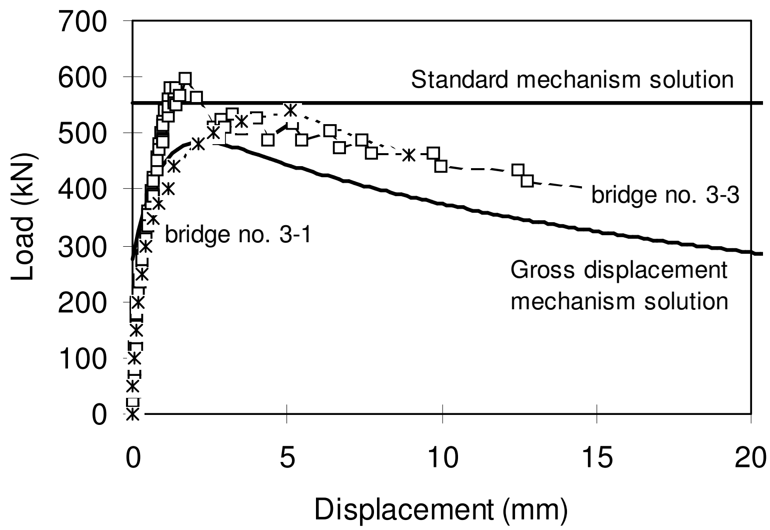

Figure B-2 Experimental and predicted load vs. displacement response of single-span bridges

Thus Figure B-2 indicates that the standard mechanism solution, which does not take gross structural movements into account, is in error by less than 15 percent. Had the build up in backfill pressures been more gradual then the error would have been greater. Nevertheless, given other uncertainties, such errors may well be considered acceptable.

B2 Active pressures

As mentioned in section 2.4, active pressures are by default ignored in RING 1.5. In the case of weak fill materials (i.e. fills with low angles of shearing resistance), active pressures may be significant. However, this is to a large extent compensated by the low passive pressures implied by ⅓K_p when weak fills are present.

B3 Unusual failure mechanisms

In RING 1.5 the presence of uniaxial backfill elements means that although it is unnecessary to specify in advance the sense of the pressures, the *magnitudes* of the pressures do need to be specified in advance. Thus, as indicated previously, in order to approximately reproduce the results from full-scale tests, horizontal passive zone restraining pressures might commonly be entered as ⅓K_p. However, RING 1.5 chooses the critical failure mechanism from a multitude of possible ones and a 4 hinge failure mechanism is by no means always identified as being critical. For example, Figure 3b shows a failure mode encountered when recently assessing a short span bridge.

Here the predicted failure mode involves sliding failures at three joints, and translation rather than rotation of a section of arch into the fill, the magnitudes of the passive restraining pressures specified correspond to those mobilised in a 4 hinge mechanism. In reality the sliding failure mode predicted would be likely to more rapidly mobilize significant passive zone soil pressures. Thus the RING 1.5 strength prediction is likely to be quite conservative.

In the future it is expected that this issue will be resolved by moving away from the current indirect modelling strategy for the soil towards instead modelling the soil explicitly (i.e. using solid elements to represent the soil material).

References

B15. Gilbert M. (1997). Gross displacement mechanism analysis of masonry bridges and tunnels. Proc.11th International Brick/Block masonry conference. Shanghai. 473-482.